

## CONTROL ALGORITHMS FOR CROSS DIRECTIONAL CONTROL: THE STATE OF THE ART

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**Abstract.** Successful control of cross directional variations in sheet properties, as with any control problem, requires consideration of suitable measurements and measurement processing, suitable actuators, and a suitable control algorithm. Also, as with any control problem, the choice of a suitable control algorithm depends upon the behavior of the process, in terms of its response to the selected actuators and the nature of the variations to be controlled. This paper discusses the requirements for suitable control algorithms. Although the discussion centers on applications to paper making, the general principles are applicable to other sheetmaking processes.

**Keywords.** Computer control; adaptive control; distributed parameter systems; optimal control; cross-direction control; paper industry.

### INTRODUCTION

It may seem trivial and obvious to define the objective of cross-machine direction (CD) control to be the minimization of variations in a sheet property, such as basis weight or moisture, in the transverse or cross-machine direction. Due to the nature of variations of sheet properties, however, the objective is worthy of more careful definition. The deviation of a sheet property,  $P$ , from its desired value,  $\bar{P}$ , may be defined as a function of position on the sheet in two dimensions:

$$\Delta P(x,y) = P(x,y) - \bar{P}(x,y) \quad (1)$$

where  $x$  represents a machine-direction (MD) coordinate and  $y$  a cross machine direction (CD) coordinate. (Note that the desired value may also be a function of position.) Due to physical considerations, it is usually possible to subdivide the variations into three components:

$$\Delta P(x,y) = \Delta P_{MD}(x) + \Delta P_{CD}(y) + \Delta P_R(x,y) \quad (2)$$

where  $\Delta P_{MD}(x)$  represents those variations which influence the property uniformly across the sheet (such as consistency upsets),  $\Delta P_{CD}(y)$  represents those variations which influence the property uniformly in the machine direction (such as a poorly adjusted slice lip), and  $\Delta P_R(x,y)$  represents the residual variations which are not uniform in either direction. It is customary (and usually justified) to assume that these components of variation are statistically independent, so that the statistical variance can be expressed as the sum

of the individual components of variance:

$$\sigma^2 = \sigma_{MD}^2 + \sigma_{CD}^2 + \sigma_R^2 \quad (3)$$

It is often observed that the three components of variance are of the same order of magnitude on an uncontrolled machine, and thus it must be concluded that all three components are equally worthy of attention in order to produce the best quality product.

Superficial consideration of the preceding concepts has led many people to assume that the objective of CD control is to minimize the pure CD component of variance  $\sigma_{CD}^2$ . This term, however, represents only the constant ("d.c.") component of CD variance. In fact, there is a whole spectrum of profile variations, ranging from slowly varying components to very short term variations. The entirety of this spectrum, save only the constant part, is represented by the variance component  $\sigma_R^2$ . The time-varying component of the profile is often declared to be an "unstable profile" and due to machine problems (Burns, 1974). Indeed, it is nearly always best to attack variations at their source, rather than to compensate for them by control. In reality, however, it is impossible to identify and correct all sources of variation, and experience indicates that the residual component of variance for basis weight and moisture on most paper machines is substantial. Therefore, a proper objective of CD control could be to minimize both the CD and residual variance, just as the objective of MD control is to minimize the variance component  $\sigma_{MD}^2$  (which represents the entire spectrum of MD variations).

The CD control problem is to control a distributed parameter process which is properly described by a partial differential equation. Even if we assume this equation to be linear with constant coefficients, the general solution using, say, a least squares optimal control approach would be very complex. In order for the control synthesis problem to be tractable we must make certain assumptions. Fortunately in many cases the following assumptions are reasonable:

1. The dynamic (temporal) response of the process is assumed to be nearly independent of cross direction position.
2. Spatial coupling or interaction in the cross direction may be treated as static (instantaneous) behavior which drives the uniform dynamics above. By spatial coupling we mean that an adjustment to an actuator at one CD position will in general cause a change in the sheet property at other CD positions (i.e., the response is distributed in space). We shall refer to assumptions 1 and 2 taken together as the "separability" property.
3. Except for boundary conditions (edge effects), the static spatial profile response to an actuator adjustment is independent of actuator location, except, of course, that the response is shifted in space. We shall refer to this assumption as the "homogeneity" property.
4. Except for boundary conditions (edge effects), the static spatial profile response to a single actuator adjustment is symmetric about the center of influence of the actuator. We shall refer to this assumption as the "symmetry" property.

CD control algorithms are distinguished from each other by their treatment of the spatial coupling and dynamics.

**THE STATIC CONTROL PROBLEM:  
SPATIAL COUPLING**

The Continuous Case

If the parameters of the distributed process are assumed "constant" and the process may be assumed to behave linearly within a reasonable operating region, then the process response may be characterized by a linear partial differential equation with constant parameters over its domain of definition. Given the assumptions from the preceding section and ignoring boundary conditions for the moment, this domain may be taken to be "double-sided" and infinite. The process behavior may then be represented by the convolution equation

$$\Delta y(\ell) = \int_{-\infty}^{\infty} \phi(\ell-v)\Delta u(v)dv + \Delta y_0(\ell) \tag{4}$$

where  $\phi(\cdot)$  is the influence function (or impulse response),  $\Delta u$  is a change in the CD control

variable,  $\Delta y$  is the corresponding change in the sheet property variable, such as basis weight.  $\Delta y_0$  is the deviation from the desired profile for  $\Delta u = 0$ .

A typical objective is

$$\min_{\Delta u(\ell)} J = \int_{-\infty}^{\infty} (q\Delta y(\ell)^2 + p\Delta u(\ell)^2)d\ell \tag{5}$$

where  $p$  and  $q$  are scalar weighting factors chosen to include a penalty on the magnitude of the control correction. The expression may be easily modified to include some other function of  $\Delta u$ , such as the second derivative to penalize bending of a slice lip, for example. The "non-causal" function  $\phi(\ell-v)$  is (following the previous assumptions) symmetric around  $v = \ell$ , and does not depend on  $\ell$ ; only on  $|\ell-v|$ .  $\Delta y(\ell)$  is the control error; i.e., the deviation from the target profile. Applying the 2-sided Laplace transformation, eq. (1) may be expressed as

$$\Delta Y(s) = \Phi(s) \Delta U(s) + \Delta Y_0(s) \tag{6}$$

The optimal control solution to this problem may be shown to be

$$\begin{aligned} \Delta U_{opt.} &= -(q\Phi(s)\Phi(-s) + p)^{-1}q\Phi(-s)\Delta y_0(s) \\ &= -G(s)\Delta y_0(s) \end{aligned} \tag{7}$$

All Laplace transforms must be treated as 2-sided transforms. The transformed adjoint of  $\phi$  is  $\phi^*(s) = \phi(-s)$ . In the case of causal systems, Eq. (7) is no solution, and spectral factorization must be utilized to obtain a realizable solution. This problem was solved more than 20 years ago in another form by S. S. L. Chang (1961), and more recently revisited by Grimble (1977) to solve the finite-domain problem.

Example 1. Consider the response function

$$\Phi(s) = \frac{1}{s+a} + \frac{1}{-s+a} = \frac{2a}{(a-s)(a+s)}$$

If  $p = 0$ ,

$$\Delta U_{opt}(s) = -\frac{a^2 - s^2}{2a}\Delta Y_0(s)$$

If  $\mathcal{L}\{\Delta y_0(\ell)\} = \Phi(s)e^{-\lambda 0^s}$ , the control will be

$$\Delta U_{opt}(s) = e^{-\lambda 0^s}$$

i.e.

$$\Delta u(\ell) = -\delta(\ell - \ell_0)$$

which is an impulse at  $l = l_0$ . Obviously, the effect of this impulse is to cancel  $\Delta y_0(l)$  exactly. For any other form of  $\Delta y_0(l)$ ,  $\Delta u(l)$  will be a continuous control.

### The Discrete Case

Most often, the control variable may only act at spatially discrete points at a uniform distance  $\Delta L$  apart. This is usually the case in CD control of basis weight when using slice screw control on paper machines.

In such a case, the system is described by

$$\Delta y(l) = \sum_{i=-\infty}^{\infty} \phi(l-l_i) \Delta u(l_i) + y_0(l) \quad (8)$$

$\Delta y(l)$  is the response to impulse controls at discrete points  $l_i = i\Delta L$ . The objective will be

$$\min J = \sum_{i=-\infty}^{\infty} (q \Delta y(l_i)^2 + p \Delta u(l_i)^2) \quad (9)$$

The solution in the  $z = e^{\Delta L s}$  domain is

$$\Delta U(z)_{\text{opt}} = - (q \Phi(z) \Phi(z^{-1}) + p)^{-1} q \Phi(z^{-1}) \Delta Y_0(z) \quad (10)$$

where the adjoint of the z-transform of  $\phi$  is  $\Phi^*(z) = \Phi(z^{-1})$ ;  $Z(\phi(l)) = \Phi(z)$ . It is assumed that only points  $l = l_i$  are "measured" in  $\Delta y(l)$ , which means that  $\Delta y$  will be represented by its sampled function which has a z-transform  $\Delta Y(z)$ .

**Example 2.** A common response characteristic for air showers on a calender stack or rewet sprays for moisture control is illustrated in discrete form in Fig. 1. In such cases the region of influence of each actuator overlaps with its neighbors on either side. If the overlap extends only to one actuator on either side, as shown in Fig. 1, the discrete response function may be represented as

$$\phi(z) = \frac{(1 + \alpha z^{-1})(1 + \alpha z)}{1 + \alpha^2} \quad (11)$$

where  $\alpha$  is related to the "coupling coefficient" ("a" in Fig. 1) by  $\alpha = a/(1 + a^2)$ .

The optimal controller transfer function with  $p = 0$  is:

$$\frac{\Delta U(z)_{\text{opt}}}{\Delta Y_0(z)} = - \frac{1}{\Phi(z)} \quad (12)$$

$$= - \frac{1 + \alpha^2}{1 - \alpha^2} (\dots - \alpha^3 z^{-3} + \alpha^2 z^{-2} - \alpha z^{-1} + 1 - \alpha z + \alpha^2 z^2 - \dots)$$

Figure 2 illustrates the optimal control weighting function for two cases: weak coupling (small values of  $\alpha$ ) and strong coupling (large values of  $\alpha$ ). This example illustrates the obvious fact that stronger spatial coupling in the process requires stronger decoupling action of the actuators. Note also that the process response function,  $\Phi$ , which might be characterized as "spatially damped" or "spatially low-pass" leads to an oscillatory control transfer function. This is analogous to the time domain situation when deadbeat control is demanded of a highly damped or low-pass process. We shall offer more observations on this subject shortly.

### The Infinite Matrix Approach

Another approach may be taken to the problem discussed above by considering the matrix representation  $K$  of the operator  $\phi(l)$ . The domain of  $K$  will be an infinite set, representing discrete points on  $(-\infty, \infty)$ . The range of  $K$  will be the measured response points on  $(-\infty, \infty)$ . Interpreting this in terms of  $\phi(l)$ , the column elements of the infinite matrix  $K$  will correspond to the terms  $\phi(l_k \pm i\Delta L)$  for elements at element numbers  $(k \pm i, k)$  of the matrix. Since this is a symmetric, stable matrix, the inverse of  $K$  will be an infinite matrix symmetric around the diagonal. These symmetric terms are equal to the weighting terms of  $\Phi(z)^{-1}$ , as in Figs. 1 and 2. Likewise, as the coupling effects become stronger, the diagonal band becomes wider, since the number of significant terms in  $\Phi(z)^{-1}$  becomes greater. The input-output relationship of the process is

$$\Delta Y = K \Delta U + \Delta Y_0 \quad (13)$$

The problem may be generalized by assuming that the relationship between actuators and measurement sample points is not one to one, so that the matrix  $K$  need not be square.

The objective is to

$$\min_{\Delta U} J = \Delta Y^T Q \Delta Y + \Delta U^T P \Delta U \quad (14)$$

where  $P$  and  $Q$  are symmetric and positive definite. The solution to this problem is

$$\Delta U_{\text{opt}} = - (K^T Q K + P)^{-1} K^T Q \Delta Y_0 \quad (15)$$

If  $Q = qI$ , and  $P = pI$ ,

$$\Delta U_{\text{opt}} = - (qK^T K + pI)^{-1} K^T q \Delta Y_0 \quad (16)$$

for  $p = 0$ ,

$$\Delta U_{\text{opt}} = - (K^T K)^{-1} K^T \Delta Y_0 = K^+ \Delta Y_0 \quad (17)$$

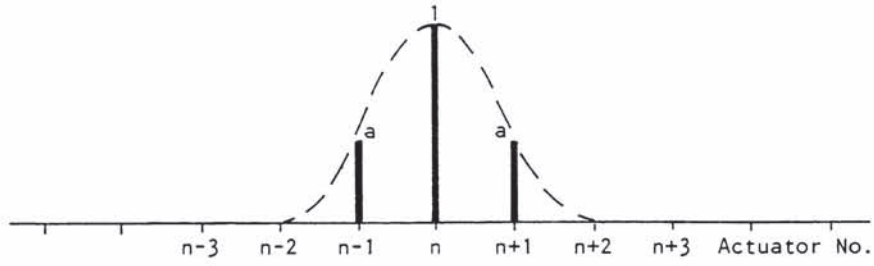


Fig. 1. Discrete response model for spatial coupling.

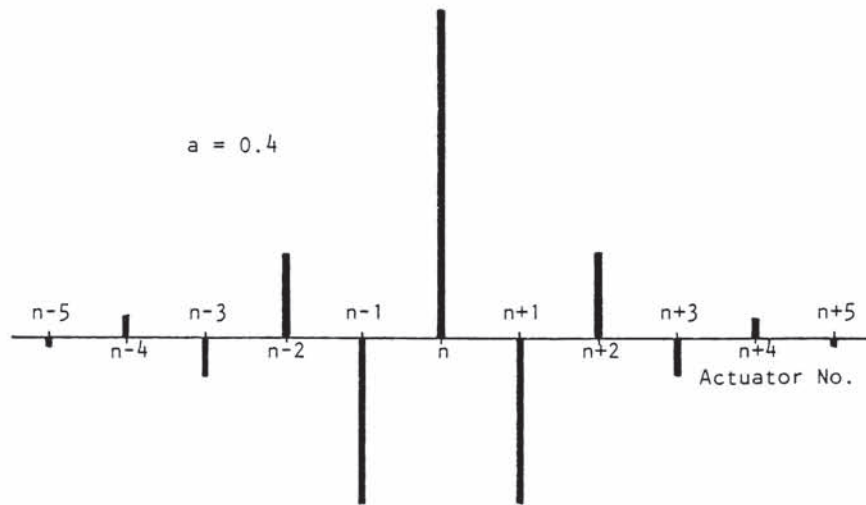
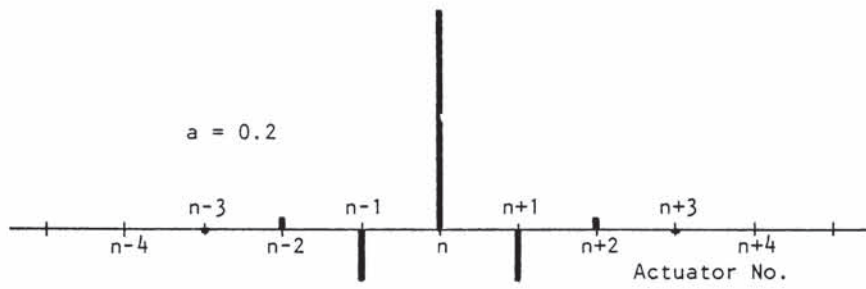


Fig. 2. Control weighting functions for process of Fig. 1

which is the straightforward least squares solution to (14) and (15) when  $p = 0$ . For square matrices  $K$ , the pseudoinverse  $K^+ = K^{-1}$ . This is the usual decoupling solution to the control problem. Observe that such a solution can be interpreted as a special case of the general least squares optimal control solution. We note also that  $P$  can easily be chosen to penalize bending (rather than deviation from a flat slice profile) which is much more appropriate to slice lip control.

### The Finite Matrix Approach

While the two-sided  $z$  transform and infinite matrix analyses shed considerable light on the nature of decoupling methods, real process systems are defined on finite intervals  $[0, L]$ . For this reason, boundary conditions (edge effects) may come into play such that corner elements of a finite matrix representation,  $K_F$ , differ from the corresponding elements in the infinite matrix,  $K$ . It is very interesting to note, however, that even if the finite matrix,  $K_F$ , is simply a truncated version of  $K$  (Fig. 3), the corner terms of the inverse,  $K_F^{-1}$ , will be different from the corresponding elements of  $K^{-1}$ . Figures 4 and 5 illustrate the finite matrix equivalents of the responses and control weighting functions previously illustrated in Fig. 2. Note that even though the matrix  $K_F$  displays homogeneous response for all actuators across the full width, the control weighting required near the edges is substantially different from that in the center, as shown by  $K_F^{-1}$ . Note also that stronger coupling causes the diagonal band to widen and elements near the corners to assume quite different values. Strong coupling causes the  $K_F$  matrix to become ill-conditioned and, in the limit, singular so that no inverse will exist. This ill-conditioning is critically dependent on  $|\operatorname{Re} \lambda_s|_{\min}$ , where  $|\lambda_s|_{\min}$  is the eigenvalue of smallest magnitude in the continuous representation. In the discrete case, the condition transforms into how close  $|\operatorname{Re} \lambda_z|_{\max}$  is to the point  $z = 1$ . Our findings show that if  $\Delta L |\operatorname{Re} \lambda_s|_{\min} \lesssim 1$ , the inverse is well-behaved in the sense that it produces a reasonably narrow diagonal band.

Simplified modeling. Figures 4 and 5 illustrate that for certain types of responses the finite matrix model and infinite matrix model (or  $z$  transform model) are nearly identical except at the corners. Thus one may characterize the entire matrix inverse by a few weighting coefficients which represent most rows, plus a few more for the corners. This avoids the necessity of storing the inverse of a large matrix, yet still allows special treatment of the edges. Good engineering judgement is required, however, when choosing the form of the response model for this scheme to work well.

Choice of actuator spacing. Strong coupling between the responses to adjacent actuators often simply represents a poor choice of actuator spacing. This is analogous to choosing too high a sampling rate in a sampled data system. If the spacing cannot be conveniently changed, the problem may often be solved by controlling groups of actuators in parallel, shutting some off, or

computing the control based upon wider spacing and using interpolation to determine the settings of the remaining actuators.

An especially critical factor to the success for a CD control is the spatial relationship between the measurements taken at the scanner and the actuators used to control the profile. Due to shrinkage, sheet wander, and the sheer physical distance between actuators and sensors, this relationship is surprisingly difficult to determine with the required precision. This "mapping" problem has been attacked by empirical means, including the application of dye trails to the sheet (McFarlin, 1983) and observing the response to large actuator perturbations. When known reference points can be established by manual or automatic measurements, much better precision can be expected. Carey and coworkers (1975) noted, for example, that the position of the wet end cutting jets on a paper machine bear a fixed relationship with slice actuator positions, and serve as a reference for the origin of the sheet edges as observed at the dry end scanner.

In addition to the penalties placed upon bending or other control actions, hard bending constraints are advisable for the slice, even though they yield sub-optimal results. Several implementers have concluded that slice lip displacement measurement is highly desirable for this reason, and essential for a self-tuning scheme (Carey and coworkers, 1975; Karlsson and Haglund, 1983; Wilhelm, 1982). The constraint on slice bending also implies the need for fail safe operation of actuators in the event of power failure, and bumpless initialization of the control when power is restored or control is turned on.

### More Complex Response Shapes

Control of CD basis weight on paper machines has long been known as an especially difficult problem. The primary reason for this difficulty is the complexity of the profile response to a slice adjustment, which generally assumes the shape shown in Fig. 6. Although evidence of this response shape was published at least as early as 1957 (Cuffey, 1957), it does not seem to have been widely known until more recently. It has now been corroborated by other researchers (Karlsson and coworkers, 1982) who have also shown that the width of the response is highly dependent upon the type of machine.

Even though the assumptions of homogeneity of response across the width of the machine and linearity within an operating range can be demonstrated on a stable machine (Karlsson and Haglund, 1983), the severity and/or width of the response shape may preclude simple decoupling solutions such as those discussed in the previous section. Figure 7 shows a finite matrix model corresponding to the response of Fig. 6 and its inverse. Recall that the inverse matrices in Figs. 4 and 5 indicated consistent decoupling weighting coefficients over most of the machine. It is clear from Fig. 7 that the finite width combined with the response shape of Fig. 6 will require quite different control action depending upon the CD

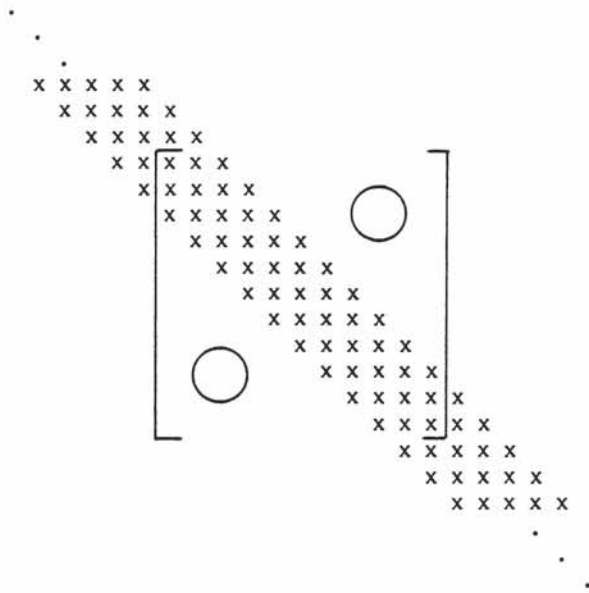


Fig. 3. Truncated infinite band diagonal matrix model.

$$\begin{bmatrix} 1.00 & 0.20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.20 & 1.00 & 0.20 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.20 & 1.00 & 0.20 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.20 & 1.00 & 0.02 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.20 & 1.00 & 0.20 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.02 & 1.00 & 0.02 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.20 & 1.00 & 0.20 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.20 & 1.00 & 0.20 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.20 & 1.00 \end{bmatrix}$$

$$\begin{bmatrix} 1.04 & -0.22 & 0.05 & -0.01 & 0 & 0 & 0 & 0 & 0 \\ -0.22 & 1.09 & -0.23 & 0.05 & -0.01 & 0 & 0 & 0 & 0 \\ 0.05 & -0.23 & 1.09 & -0.23 & 0.05 & -0.01 & 0 & 0 & 0 \\ -0.01 & 0.05 & -0.23 & 1.09 & -0.23 & 0.05 & -0.01 & 0 & 0 \\ 0 & -0.01 & 0.05 & -0.23 & 1.09 & -0.23 & 0.05 & -0.01 & 0 \\ 0 & 0 & -0.01 & 0.05 & -0.23 & 1.09 & -0.23 & 0.05 & -0.01 \\ 0 & 0 & 0 & -0.01 & 0.05 & -0.23 & 1.09 & -0.23 & 0.05 \\ 0 & 0 & 0 & 0 & -0.01 & 0.05 & -0.23 & 1.09 & -0.22 \\ 0 & 0 & 0 & 0 & 0 & -0.01 & 0.05 & -0.22 & 1.04 \end{bmatrix}$$

Fig. 4. Finite matrix model and inverse for weak coupling ( $a = 0.2$ ).

$$\begin{bmatrix} 1.00 & 0.40 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.40 & 1.00 & 0.40 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.40 & 1.00 & 0.40 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.40 & 1.00 & 0.40 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.40 & 1.00 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.40 & 1.00 & 0.40 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.40 & 1.00 & 0.40 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.40 & 1.00 & 0.40 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.40 & 1.00 \end{bmatrix}$$

$$\begin{bmatrix} 1.25 & -0.62 & 0.31 & -0.16 & 0.08 & -0.04 & 0.02 & -0.01 & 0 \\ -0.62 & 1.56 & -0.78 & 0.39 & -0.20 & 0.10 & -0.05 & 0.02 & -0.01 \\ 0.31 & -0.78 & 1.64 & -0.82 & 0.41 & -0.20 & 0.10 & -0.05 & 0.02 \\ -0.16 & 0.39 & -0.82 & 1.66 & -0.82 & 0.41 & -0.20 & 0.10 & -0.04 \\ 0.08 & -0.20 & 0.41 & -0.83 & 1.66 & -0.83 & 0.41 & -0.20 & 0.08 \\ -0.04 & 0.10 & -0.20 & 0.41 & -0.83 & 1.66 & -0.82 & 0.39 & -0.16 \\ 0.02 & -0.05 & 0.10 & -0.20 & 0.41 & -0.82 & 1.64 & -0.78 & 0.31 \\ -0.01 & 0.02 & -0.05 & 0.10 & -0.20 & 0.39 & -0.78 & 1.56 & -0.62 \\ 0 & -0.01 & 0.02 & -0.04 & 0.08 & -0.16 & 0.31 & -0.62 & 1.25 \end{bmatrix}$$

Fig. 5. Finite matrix model and inverse for stronger coupling ( $a = 0.4$ ).

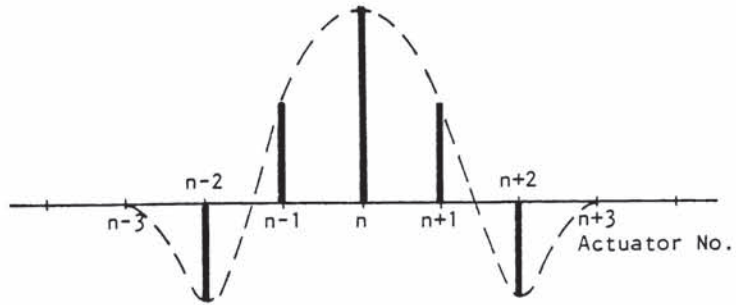


Fig. 6. Response of basis weight profile to a slice screw adjustment.

1.00	0.50	-0.50	0	0	0	0	0	0
0.50	1.00	0.50	-0.50	0	0	0	0	0
-0.50	0.50	1.00	0.50	-0.50	0	0	0	0
0	-0.50	0.50	1.00	0.50	-0.50	0	0	0
0	0	-0.50	0.50	1.00	0.50	-0.50	0	0
0	0	0	-0.50	0.50	1.00	0.50	-0.50	0
0	0	0	0	-0.50	0.50	1.00	0.50	-0.50
0	0	0	0	0	-0.50	0.50	1.00	0.50
0	0	0	0	0	0	-0.50	0.50	1.00

0.27	0.67	-0.79	0.82	-0.36	-0.18	0.71	-0.83	0.77
0.67	-0.26	1.08	-0.78	0.44	0.22	-0.76	0.90	-0.83
-0.79	1.08	-0.50	0.86	-0.28	-0.14	0.66	-0.76	0.71
0.82	-0.78	0.86	0.12	0.24	0.12	-0.14	0.22	-0.18
-0.36	0.44	-0.28	0.24	0.48	0.24	-0.28	0.44	-0.36
-0.18	0.22	-0.14	0.12	0.24	0.12	0.86	-0.78	0.82
0.71	-0.76	0.66	-0.14	-0.28	0.86	-0.50	1.08	-0.79
-0.83	0.90	-0.76	0.22	0.44	-0.78	1.08	-0.26	0.67
0.77	-0.83	0.71	-0.18	-0.36	0.82	-0.79	0.67	0.27

Fig. 7. Finite matrix model and inverse for response of Fig. 6.

location of a profile deviation. Note particularly that this is true even though the process response was assumed consistent all the way to the edges of the machine. While papermakers often express concern that control strategies should account for wave reflections at the edge of the machine, the authors' experience indicates that the phenomenon illustrated by Fig. 7 may be a more relevant explanation of the machine tender's observation that the profile responds differently at the edges. Indeed, it may not be that the response is so different, but simply that the necessary action required to decouple is different. At the same time, it is easy to model wave reflections in a finite matrix model, so, in fact, both phenomena may be accounted for.

The implications of the phenomena discussed above for control are quite strong: when complex response shapes or very strong coupling are combined with a finite machine width, the inverse z-transform solution, using a uniform control weighting across the machine, is inappropriate. The boundary conditions require that a (large) set of linear equations, represented by Eq. 18, be solved to determine the decoupled control action.

$$\begin{aligned}\Delta u_{\text{opt.}} &= -(K_F^T K_F)^{-1} K_F^T \Delta y_0 \\ &= -K_F^+ \Delta y_0\end{aligned}\quad (18)$$

Although the number of equations (corresponding to the number of actuators across the machine) may be large (up to 100 or so), the equation may be rearranged so that no matrix inversion is required, and efficient methods are available to take advantage of the symmetry and band diagonal nature of  $K_F$ .

#### Application of Self Tuning to the Static Problem

Besides having a complex shape, the basis weight profile response changes with different grades and operating conditions, and is especially dependent upon machine speed, since this changes the time available for wave propagation across the sheet. Furthermore, it is a tedious job to determine the response from empirical data, given that it must be extracted from data containing all CD and MD disturbances. For this reason some systems have incorporated parameter identification packages to determine the response model on-line (Karlsson and Haglund, 1983; Wilhelm, 1982), and some use the parameter identification continuously to form a self-tuning regulator operating in the space domain. Such a scheme helps to accommodate step-wise changes in the response, such as occur at a grade change, or long-term changes due to changing machine condition. The principles of recursive parameter estimation schemes for parameter identification are widely known in the literature on self-tuning regulators (see, for example Clarke and Gawthrop, 1979; Åström, 1980; or Isermann, 1982), and can be easily adapted to the spatial response models. It is also possible to formulate a hybrid "recursive batch" form of extended least squares identification, in which the multiple control corrections made at

any one point in time are treated as a new batch of data to be combined with all previous batches. Since the solution of the control Eq. (18) also corresponds to solving a least squares problem, much of the software can be shared between the identification and control problems.

#### Static Interactions Among Properties

Multiple profiles influenced by the same actuators. Changing the CD profile of one sheet property may cause a significant change in another. For example, changes in the basis weight profile clearly cause nearly proportional changes in caliper, and since they redistribute the CD load profile on the driers and drainage section they also strongly influence moisture. To the extent that these interactions are linear, we can expect the shape of the static response of each affected variable to the CD actuators to be the same, only scaled by their relative static gains.

Under ideal circumstances it might be supposed that the best basis weight CD profile would produce the best moisture or caliper profiles, but since there are other CD disturbances affecting those variables this is not generally the case. Thus, if no CD control exists for moisture or caliper, or if their control actuators are easily outranged, it may be desirable to sacrifice the quality of the basis weight CD profile in order to improve the quality of the moisture or caliper CD profile. Fortunately this requires only a simple extension of Eqs. (14) and (15). The cost function to be minimized can be extended to include an additional CD profile. Let the new CD variable be  $z$ , and since its CD response to  $\Delta u$  is proportional to that of  $y$  we may write (recalling Eq. (13)):

$$\Delta z = cK\Delta u + \Delta z_0 \quad (19)$$

where  $c$  is a scalar constant of proportionality. Then we can extend the quadratic cost criterion (14) to

$$J' = \Delta y^T Q \Delta y + \Delta u^T P \Delta u + \Delta z^T R \Delta z \quad (20)$$

where  $R$  is symmetric and positive definite. The control which minimizes  $J'$  is then:

$$\Delta u_{\text{opt}} = -(K^T(Q+c^2R)K + P)^{-1}K^T(Q\Delta y_0 + cR\Delta z_0) \quad (21)$$

Now if we define

$$Q_\lambda = \lambda Q \quad (22a)$$

and

$$R_\lambda = \frac{1-\lambda}{c} Q \quad (22b)$$

then

$$\Delta u_{\text{opt.}} = -(K^T Q_\lambda K + P)^{-1} K^T Q_\lambda (\lambda \Delta y_0 + \frac{1-\lambda}{c} \Delta z_0) \quad (23)$$

which has exactly the same form as Eq. (15), but uses a weighted sum of the error profiles  $\Delta y_0$  and  $\Delta z_0$  instead of  $\Delta y_0$  alone. The scalar weighting factor may be changed by an operator at will to produce the trade-off desired. Equation (21) can also form the basis of more complex schemes in which individual diagonal elements of Q and R could be changed to effect the trade-off between two profiles only in selected regions (see next section). This method is closely related to the scheme discussed by Boyle (1978), although he was not concerned with trade-offs between two variables.

Multiple actuators for the same profile. When more than one set of CD actuators is available to influence the same sheet property, the control flexibility can be increased. For example, if sectionalized steam boxes are available for moisture control, the basis weight/moisture trade-offs discussed above need only be brought into play when certain sections of the steam boxes are outraged by a large moisture profile disturbance. McFarlin (1983) discusses other examples, including the use of a sectionalized dryer hood on a Yankee dryer as a coarse actuator to keep steam box sections within limits.

Similarly, MD and CD actuators influencing the same property can easily be coordinated to achieve optimal results. For example, in order to minimize energy consumption when utilizing remoisturizing sprays, dryer pressure may be adjusted so that at least one remoisturizing spray is shut off completely. Conversely, McFarlin (1983) points out that when using steam boxes, maintaining at least one section fully open will maximize throughput on a dryer-limited machine. When the CD actuators have limited range, it may be more important to coordinate the MD actuator to keep them midranged for greatest effectiveness. Boyle (1978) included such a criterion directly in a quadratic objective function, but midranging is just as easily overlaid on the CD control without affecting its optimality.

#### DYNAMIC ASPECTS OF CD CONTROL

The requirements for dynamic compensation in CD control vary widely with the application, but are dictated by the same considerations as any other feedback control loop. Among these considerations are:

- the dynamic response of the process to changes in the actuators;
- dynamic response of the actuators themselves;

-transport delay between actuator and measurement;

-filtering required to eliminate "noise" from the measurement;

-the stochastic characteristics of the disturbances to be controlled.

Measurement filtering is important in most CD control applications, since the profile measurement is generally derived from a point sensor scanning back and forth across the sheet. Thus, in any single scan both MD and CD variations appear. Since MD variations are strictly noise to the CD control loop, we must either perform low pass filtering or tune the CD control loop to act as its own low pass filter. The most appropriate choice depends upon other factors, as we shall see. In many cases it is useful to provide some logic to ignore certain scans where MD variations are believed to be strong. There is also considerable room for future applications of more advanced estimation and filtering techniques to the separation of MD and CD information.

The dynamic response of caliper (controlled by air showers at the calender stack) is dominated by the heat transfer dynamics, with negligible transport delay. For such a loop, it is clear that a PI or PID control algorithm will do nicely (Fjeld and Hickey, 1981), where the proportional and derivative terms are used to achieve faster response, and integral action is required to eliminate the static profile disturbance. It makes little sense to heavily filter the profile for this loop, thus adding negative phase shift which destabilizes the loop, since the proportional and derivative action will negate its effect anyway. Instead, if MD variations pose a problem, one can detune the PID control to achieve the effective filtering required, or provide some logic to inhibit CD control when MD variations are large.

At the other end of the spectrum is basis weight CD control, for which the process dynamics are nearly pure transport delay, although the slice actuators may contribute something like a first order lag if they are slow to respond, such as thermal actuators. In the case where the transport delay is clearly dominant, the simplest approach is to allow a full transport delay plus one full scan of the traversing measurement to elapse between control corrections. This method allows deadbeat response if the static model is correct, and may be entirely adequate given the region of the spectrum we are trying to control. In order to avoid aliasing, due to the longer sampling period, some profile filtering is required in this case, and does little to detract from performance. On the other hand, if the actuator response is particularly slow, more complex dynamic compensation may be required, utilizing some form of deadtime compensation. Tong (1979) proposed such a scheme in a state variable formation, and McFarlin (1983) has hinted of an application.

Dynamic Interactions Among Variables. In our discussion of the static control problem we noted

that more than one CD property may be influenced by the same set of actuators, and discussed a method for trading off control objectives for more than one property. If each property is independently being controlled by a different set of actuators, however, the problem becomes the typical interacting multivariable dynamic system, made more complex by the distributed nature of the CD variables. Fortunately, the coupling between any two variables is not usually bidirectional (e.g., the slice influences both weight and caliper, but the caliper actuators do not usually influence weight; the slice influences both bone dry weight and moisture, but the moisture actuators do not influence bone dry weight). Thus, in most cases, full scale decoupling is not required. Feedforward compensation between the loops is likely to be fully adequate. Of course, the simplest way to achieve dynamic decoupling is to tune interacting loops so that one is significantly faster than the other. Based upon the lack of discussion on decoupling in the literature, it would appear that the latter method is considered adequate in most present applications.

#### CONCLUSIONS

Depending upon the process, successful cross direction control can be very easy or very difficult to achieve. In the absence of spatial coupling (but assuming that the actuators are close enough that there are no uncontrollable voids), the problem reduces to a set of parallel independent control loops which can most likely be identically tuned. Compensation for process dynamics may be accomplished using the standard techniques applied to other single-input-single-output control problems. When spatial coupling exists, the present state of the art requires that the spatial (static) and temporal (dynamic) problems be separable. In that case, the complexity of the techniques required for treating the spatial coupling depends upon the strength and shape of the coupling, as well as the influence of boundary (edge) conditions. Providing spatial decoupling in the control algorithm improves the speed of response to disturbances, and may improve the stability of the temporal part of the control as well.

On the other hand, good engineering judgement is required when providing decoupling. If the original process coupling is very strong, the unconstrained control solutions will be unreasonable. Since ordinary feedback control will arrive at the same solution, decoupling itself is not the problem, but the spatial bandwidth of the CD disturbances to be controlled must be commensurate with the means used for control. A number of techniques are available for achieving this match, including quadratic penalties on control amplitude or its derivatives (to penalize bending), changing the spatial sampling interval by combining or eliminating actuators, and spatial low pass filtering of the profile to be controlled.

In cases where the process spatial response changes with operating conditions, self-tuning schemes based upon on-line identification may be

used. This technique can also be used to minimize the laborious effort associated with determining response, even if the response is constant over time.

Practical schemes have also been applied to allow trade-offs between CD profiles of different properties affected by the same actuators, and we may expect to see more attention focused upon dynamic interactions between different CD control loops in the future. In general, appreciation for the importance of CD control has been building for some time, and recent advances in our understanding of the problem have combined with affordable computer power to produce rapid progress on the solutions in the past five years. Given the untapped market and high level of interest, we may expect many more exciting technical developments to evolve in the next few years.

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